NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 7
COURSE CODE: RAN701S	**COURSE NAME:** REAL ANALYSIS
SESSION: JULY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	DR. NA CHERE
MODERATOR:	PROF. F MASSAMBA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. Number the answers clearly.
4. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [16]

Use the definition of the limit of a sequence to establish the following limits.

1.1. $\lim_{n\to\infty}\left(\frac{2}{\sqrt[3]{n+1}}\right) = 0.$ [8]

1.2. If $\lim_{n\to\infty}(x_n) = 3$, then $\lim_{n\to\infty}\left(\frac{3x_n-1}{4}\right) = 2.$ [8]

QUESTION 2 [14]

Determine whether each of the following sequences converges or diverges.

2.1. $\left(\sqrt{n^2+n}-n\right).$ [8]

2.2. $\left((-1)^n\frac{n-3}{n}\right)$ [6]

QUESTION 3 [10]

Prove that $\lim_{n\to\infty}(x_n) = 0$ if and only if $\lim_{n\to\infty}(|x_n|) = 0$. Give an example to show that the convergence of $(|x_n|)$ need not imply the convergence of (x_n).

QUESTION 4 [15]

4.1. Determine whether the series $\sum_{n=1}^{\infty}\frac{n^n}{n!}$ converges or diverges. [7]

4.2. Determine whether the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{3^n+4^n}$ converges absolutely or conditionally. [8]

Question 5 [10]

Show that the sequence $(x_n) = \left(\frac{3}{\sqrt{n}}\right)$ is a Cauchy sequence. [10]

QUESTION 6 [9]

6.1. Define what does it mean to say a sequence (x_n) in $\mathbb{R}$ is increasing? [2]

6.2. Let $x_1 = 1$, $x_{n+1} = \sqrt{2+x_n}$ for $n \in \mathbb{N}$. Show that (x_n) is increasing. [7]

QUESTION 7 [18]

Let $A \subseteq \mathbb{R}$ and let $f: A \to \mathbb{R}$.

7.1. Define what does it mean to say f is uniformly continuous on A? [3]

7.2. Let $f(x) = x^2$.

(a) Use the definition of uniform continuity to show that f is uniformly continuous on $[-4, 2]$.

[7]

(b) Use the nonuniform criteria to show that f is not uniformly continuous on $(-\infty, \infty)$. [8]

QUESTION 8 [8]

Apply the mean value theorem to prove that $|\ln y - \ln x| \leq 4|y - x|$ for $x < y$ and

$x, y \in [\frac{1}{4}, 4]$.

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